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OPTIMAL CONTROL OF THE WHEELCHAIR WHEELIE

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ABSTRACT
The wheelchair wheelie is a maneuver employed to overcome obstacles and descend ramps, for instance. The task is similar to the stabilization problem of an inverted pendulum that is extensively described in the control theory literature. However, in this case, the goal is to maintain the user and the wheelchair in equilibrium on wheels, which is achieved when the center of mass of the system is aligned with the rear axle in the vertical direction. This work investigates a controller to perform the wheelie in power-assisted wheelchairs using optimal control theory and a model of the user and wheelchair system. The proposed approach leads to a controller capable of rising the wheelchair, which is able to reject perturbations and which is robust to typical parameter uncertainties.

KEY WORDS
Wheelie, wheelchair, optimal control

1. Introduction

According to Copper *et al.* [1], manual wheelchairs are commonly used by people with lower limb disability, such as spinal cord injury, lower limb amputations or stroke. As reported in 2010 by *Instituto Brasileiro de Geografia e Estatística* (IBGE), there are 45.6 million Brazilians with some kind of disability. Among them, 7% have motor disabilities and are users or potential users of wheelchairs. In daily life, many wheelchair users suffer upper limbs injuries and face difficulties such as excessive effort during wheelchair propulsion on inclined surfaces and obstacle negotiation [2-10].

In order to reduce some of these difficulties, power-assisted wheelchairs have been proposed [4-10]. These wheelchairs are equipped with motors attached to the wheels which assist the user during manual propulsion rather than completely replace the manual propulsion as occurs with the electric wheelchair. Most power-assisted wheelchairs do not have a control system with feedback sensors and are not able to perform or sustain the wheelie, a maneuver consisting of balancing the wheelchair on two wheels (Figure 1). This maneuver is important for active users because it allows surpassing obstacles such as sidewalks or uneven terrain. Kirby et al. [11], for instance, explain in details the importance of the wheelchair wheelie in different situations.

Recent studies in the literature propose control systems and strategies to permit the wheelie in power-assisted wheelchairs. Sehoon et al. [12], for instance, propose a controller able to identify the environment and operate in adverse situations, such as overturning and the support of the propulsion on inclined surfaces. Takahashi *et al.* developed a power-assisted wheelchair to climb steps and maintain its equilibrium during this task [2-7]. In addition, the wheelchair has a system to shift the center of mass to perform the task. A motor shifts the wheelchair backwards, which results in the center of mass closer to the rear wheel, easing the lift of the front wheels. The control system is only active when the wheelchair is on two wheels [4-10].

In this context, the aim of this paper is to propose a control strategy to initiate and sustain the wheelie in power-assisted wheelchairs using an optimal control formulation and a model of the user and wheelchair system. The system requirements are investigated on the light of optimal rising patterns and the robustness of the controller to parameter uncertainties and to a typical perturbation is verified.

2. Wheelchair-User Model

2.1 Mechanical Model

When the four wheels of the wheelchair are on the ground, the system is stable and the model is composed of two bodies, the rear wheels and the rigid body containing the user and the wheelchair without its rear wheels. If there is no rear wheel slip, this model has one degree of freedom (Figure 1, left). This phase is referred to as phase 1. The wheelie requires rising the front wheel from the ground and balancing the wheelchair on the two rear wheels (Figure 1, right). In this phase, named phase 2, the model is composed of the same two bodies but it has two degrees of freedom. In this phase, the system is unstable.

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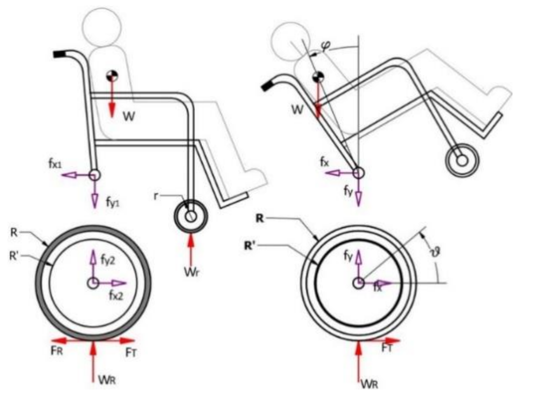

Figure 1. Model of the wheelchair-user system in phase 1 on the left and 2 on the right.

The equation of motion governing the dynamics of the model in phase 1 is

$$\frac{\tau}{R} = \left(M + \frac{J_R}{R^2}\right) \cdot \ddot{x} + F_R \tag{1}$$

where τ is the moment applied by the user and/or the assistance motor, M is the total mass of the system, R is the rear wheel radius, x is the forward displacement of the wheelchair, F_R is the rolling resistance force, and J_R is the moment of inertia of both rear wheels. Equation 1 is valid for the wheelchair on an even surface.

In phase 2, the system is an analog to the inverted pendulum on a cart. The equations of motion governing the model dynamics in this phase are

$$\begin{aligned} \tau - F_R \cdot R = {} & [J_R + (M_r + M_c) \cdot R^2] \cdot \ddot{\theta} + (M_c \cdot R \cdot l \cdot \cos\varphi) \cdot \ddot{\varphi} \\ & - M_c \cdot R \cdot l \cdot \dot{\varphi}^2 \cdot \sin\varphi \\ -\tau = {} & (M_c \cdot l \cdot R \cdot \cos\varphi) \cdot \ddot{\theta} + (J_c + M_c \cdot l^2) \cdot \ddot{\varphi} - M_c \cdot g \cdot l \cdot \sin\varphi \end{aligned} \tag{2}$$

where θ is the rear wheel angular displacement, φ is the upper body (user + wheelchair) angle with the vertical (Figure 1, right), M_r is the mass of both rear wheels, M_c is the mass of the upper body user + wheelchair without rear wheels), J_R is the moment of inertia of both rear wheels with respect to the wheels axle, J_c is the moment of inertia of the upper body with respect to its center of mass, g is the gravity acceleration and l is the distance between the rear wheels' axle and the upper body's center of mass.

The rolling resistance force is the main opposing force according to Brubaker [13] and depends on many factors, including the normal load on the tire, the tire radial stiffness, the wheel radius and the inflation pressure [14]. According to [12], the rolling resistance force can be expressed as a function of the normal force N as

$$F_R = \frac{\mu}{R} N \tag{3}$$

where μ is the friction coefficient and R is the rear wheel radius.

We have already shown the equations of motion for phase 1, Eq. (1), in which the wheelchair is stable with four wheels on the ground, and for phase 2, when wheelchair is on two wheels, Eq. (2). It is useful, for motor and controller design purposes, to determine the conditions that set the transition from one phase to the other. The transition occurs when the normal force on the front wheels reaches null in phase 1, which corresponds to the eminence of lifting off.

In order to determine the wheel torque necessary to lift off the front wheels, we first computed the forward acceleration that leads to a null normal force at the front wheels as

$$\ddot{x}_{nf} = \frac{d_{xcg}}{h_{cg}} \cdot g, \tag{4}$$

where d_{xcg} is the distance between the rear wheel axle and the center of mass of the user + wheelchair, and h_{cg} is the height of the center of mass of the user + wheelchair with respect to the ground. Plugging Eq. (4) into Eq. (1) yields

$$\tau_{nf} = \left[\left(M + \frac{J_R}{R^2}\right) \cdot \frac{d_{xcg}}{h_{cg}} \cdot g + F_R\right] \cdot R\,, \tag{5}$$

which computes the torque necessary to lift off the front wheels from the floor and initiate phase 2.

2.2 Model parameters

In this study, we employed wheelchair parameters measured from a real wheelchair available in the market (Ágile 2009, Ortopedia Jaguaribe Indústria e Comércio). As most parameters appearing in Eqs. (1), (2) and (4) are not provided by the manufacturer, it was necessary to estimate them experimentally.

The moments of inertia of the wheels with respect to its axis and of the wheelchair with respect to its center of mass were estimated using oscillation experiments [15]. The data were collect using a tachometer (Hohner Eletrônica Ltda., Artur Nogueira-SP) connected to the rear axle and a data acquisition board (NI PCI-6221 37pin, National Instruments). The measured moment of inertia of the rear wheel is 0.140 kg.m^2 and the moment of inertia of the wheelchair without wheels with respect to its center of mass is 1.67 kg.m^2.

The rear wheel mass was measured as 2.546 kg and the wheelchair mass without rear wheels was measured as 12.71 kg. The location of the wheelchair's center of mass was estimated by suspending the wheelchair from three different positions.

In order to calculate the mass, moment of inertia and center of mass location of the human body in the seated position, we used normative, anthropometric relationships reported in [16] as functions of user stature and total mass.

The rolling resistance force F_R was neglected and set to 0. All these data were used to compute the parameters of the bodies in Eqs. (1), (2) and (4). The parameter values computed for a user mass of 75 kg, a user stature of 1.75 m, and the user shoulder vertically aligned with the rear wheel axle, adopted in this study as the nominal condition (mass of 75 kg, stature of 1.75 m and horizontal axle-shoulder distance of 0), are reported in table 1.

Table 1
Parameters used for the nominal condition

Parameter	Value	Parameter	Value
M	87.7 kg	R	0.305 m
M_c	12.7 kg	d_{xcg}	0.107 m
M_r	2.54 kg	h_{cg}	0.436 m
J_R	0.14 kg.m^2	$\ddot{x}_{nf}$	1.57 m/s^2
J_c	1.67 kg.m^2	τ_{nf}	45.1 N

3. Methods

3.1 Optimal Control Formulation

There are infinite torque histories that can bring the wheelchair from lift off to the unstable equilibrium position in wheelie. In order to determine a reference, optimal trajectory in terms of minimum motor effort, an open-loop optimal control problem is formulated and solved for the nominal system parameters (user's mass of 75 kg, user's stature of 1.75 m and user's shoulder vertically aligned with rear wheel axle).

The optimal control problem consists of searching for the time histories of the system states, $\theta(t)$, $\dot{\theta}(t)$, $\varphi(t)$, $\dot{\varphi}(t)$, the control (rear wheel torque), $\tau(t)$, and the final time, tf, that minimize the cost function

$$J = \int_0^{tf} \tau^2 \, dt \,, \tag{6}$$

representing the motor effort, subject to the equations of motion in phase 2, Eq. (2), to the boundary conditions: $\theta(0) = 0$, $\dot{\theta}(0) = 0$, $\varphi(0) = \varphi_0$, $\dot{\varphi}(0) = 0$, $\dot{\theta}(tf) = 0$, $\varphi(tf) = 0$, and $\dot{\varphi}(0) = 0$, and to the upper bound of 5 s on tf, where φ_0 is the initial upper body angle with the vertical as defined in Figure 1, with all four wheels on the ground.

This optimal control problem was solve using the commercial optimal control package PROPT (Tomlab Optimization Inc.) which implements a pseudo-spectral direct collocation method and the large-scale optimization package SNOPT (Tomlab Optimization Inc.).

3.2 Controller

The inspiration of the control law is the state feedback

$$\tau = -k \cdot x, \tag{7}$$

and its determination through the solution of the problem of Linear Quadratic Regulation (LQR). Figure 2 shows a schematic diagram of the system with the control law in state space representation, where the state vector

$$x = [\varphi \quad \dot{\varphi} \quad \dot{\theta}] \tag{8}$$

represents the state variables of the system,

$$x_r = [\varphi_r \quad \dot{\varphi}_r \quad \dot{\theta}_r] \tag{9}$$

is the reference vector, τ is the system's control (rear wheel torque) and d is the disturbance.

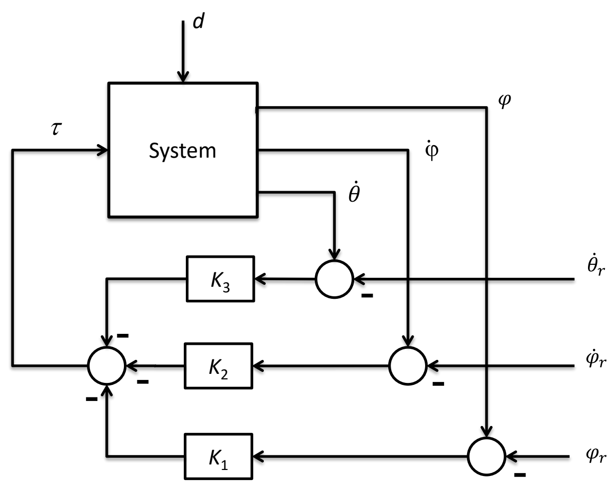

Figure 2. State Feedback.

Because the linearized dynamic model is approximately the same as the nonlinear model as φ and $\dot{\varphi}$ are small, the adopted control law tends to have good properties like those of the LQR. The linear quadratic regulator gives the system large gain and phase margins, which generally implies robust stability. The control law also provides tolerance to calibration errors of the φ angle sensor. One concern of regulatory control is precisely the calibration of the sensors and its implications on system performance. Note that one of the objectives of this control system is to ensure that φ is approximately null during the unstable equilibrium configuration. Depending on the control strategy adopted, an angular offset arising from a sensor calibration error would require a constant acceleration response of the system to balance the wheelchair on two wheels with the center not vertically aligned with the rear wheel axle. The strategy adopted tolerates this kind of sensor error because the angular velocity $\dot{\theta}$ is a state of the system. Because the angular velocity of the rear wheels follows a constant or null reference, the stable solution is associated with the actual vertical angle φ, regardless of small errors in the sensor calibration. This good property can be verified by the steady gain between φ_r and φ which results null.

Although the system tends to have a good robust stability, the difference between the nominal plant model used to design the control law and the model with actual parameters causes an increase in the relative control effort, which is directly proportional to that difference.

Optimal controller gains in Eq. (7) were determined using the optimal control problem formulation described in the previous section, with the difference that the control τ is computed as in Eq. (7), so that the only optimization parameters are the three controller parameters K1, K2 and K3. The resulting optimal values are K1 = -346.42, K2 = -105.44 and K3 = -7.6063.

5. Results

5.1 Lift off torque

Equation (5) provides the minimum rear wheel torques required to lift the wheelchair and initiate phase 2. This equation was used to construct a diagram, Figure 3, that shows the necessary wheelchair lift off torques for different combinations of user's stature and horizontal distance between the shoulder joint and the rear wheel axle. The user's mass was computed from user's stature for a Body Mass Index (BMI) of 25. The position of the shoulder with respect to the rear wheel axle can be adjusted in many wheelchairs and is an important parameter for the wheelie maneuver. In fact, the diagram shows that the lift off torques are very sensitive to this adjustment. An offset of only 5 cm is sufficient to increase the lift off torque by as much as 20 N.m.

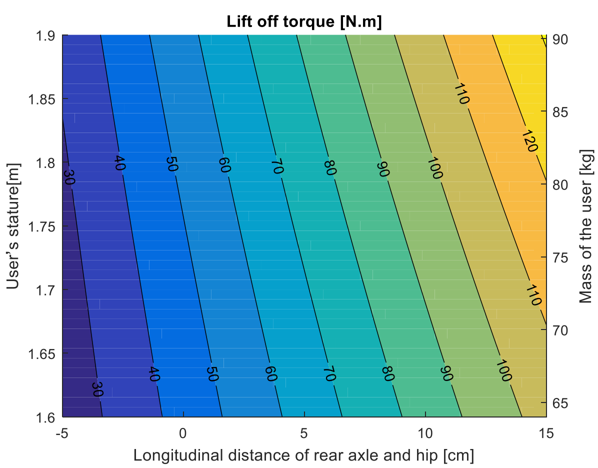

Figure 3. Influence of the user's stature, user's mass and horizontal wheel-shoulder distance on wheelchair lift off torque.

5.2 Open-loop optimal patterns

The optimal patterns obtained by solving the open-loop optimal control problem for the rising phase with minimum motor effort and for the nominal user mass and stature (75 kg and 1.75 m) is shown in Figure 4. The initial condition corresponds to the wheelchair resting with the front wheels in contact with the ground. The results show that, although it took 5 s for the wheelchair to achieve exactly the specified final boundary conditions ($\varphi = \dot{\varphi} = \dot{\theta} = 0$), the wheelchair rising movement occurred in less than a second, with the remaining time used for stabilization. Note that the maximum torque occurs at the beginning of the wheelie maneuver and achieves about 90 N.m. After this initial large torque, it falls sharply as the system's center of mass aligns vertically with the rear wheel axle. A horizontal displacement of the wheelchair occurs with a nearly complete rear wheel turn. The cost function value representing the control effort to complete the wheelie maneuver is J = 964.3 N^2.s.

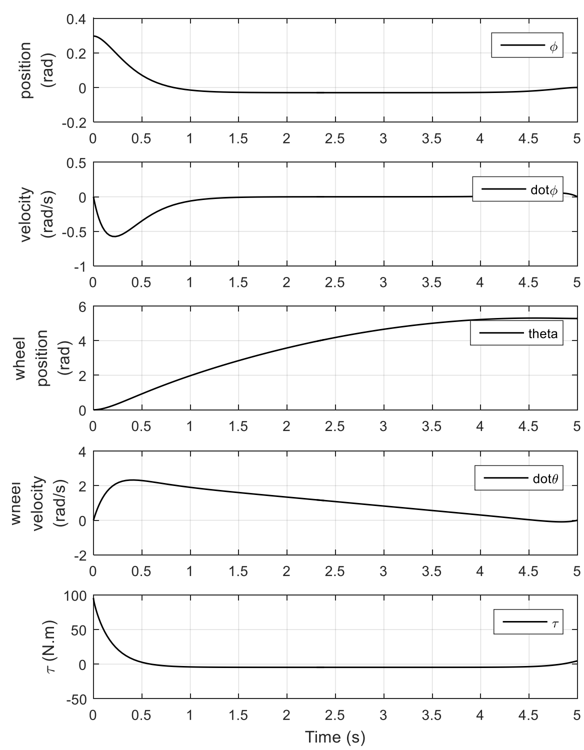

Figure 4. Open-loop optimal patterns for nominal system parameters (mass of 75 kg, stature of 1.75 m and horizontal axle-shoulder distance of 0).

5.3 Closed-loop system response

The system response for the nominal condition is depicted in Figures. 5, 6 and 7 by the bold solid line. The first 5 s correspond to the rising phase, in which the wheelchair rises from an initial condition for the front wheels on the ground to the equilibrium, upwards condition. At instant t = 5 s, the reference rear wheel angular velocity $\dot{\theta}_r$ is set to 1 rad/s. The system response to this change in the reference velocity is shown in the interval between t = 5 s and t = 10 s. At the instant t = 10 s, the system is subject to an impulsive disturbance that reproduces a horizontal force of 10 N applied 1 m above the rear wheel axle with a duration of 0.1 s. The system's disturbance rejection dynamics is shown in the interval between t = 10 s and t = 15 s.

Note that the system response in the rising phase (Figures 5, 6 and 7), is similar to the optimal patterns (Figure 4) computed by solving the open-loop optimal control problem. In fact, the motor effort simulated during the rising phase for the closed-loop system lead to a cost function value of J = 1000.2 N2.s, which is just marginally larger than the control effort obtained by solving the open-loop optimal control problem (J = 964.3 N2.s).

The robustness of the controller to some of the main system parameter uncertainties was assessed by simulating the system response to variations of user mass (Figure 5), stature (Figure 6) and horizontal distance between user's shoulder and wheel axle (Figure 7). Figure 5 shows the closed-loop system response for different values of user's mass: 60 kg, 75 kg (nominal) and 90 kg, with the other parameter values kept on their nominal values. Figure 6 shows the closed-loop system response for different values of user's stature: 1.60 m, 1.75 m (nominal) and 1.90 m., with the other parameter values kept on their nominal values. Figure 7, in turn, shows the closed-loop system response for different values of the horizontal distance of the user's shoulder with respect to rear wheel axle: -0.05 m, 0.00 m (nominal), 0.05 m and 0.10 m, with the other parameter values kept on their nominal values.

In addition to enabling the regulation of the state variables x and asymptotic tracking wheel reference speed $\dot{\theta}_r$, the system can also reduce the effects of external disturbance d. Assume, for example, an impulsive disturbance that reproduces a horizontal force of 10 N applied 1 m above the center of mass along 0.1 s. The graphs in figure 5, 6 and 7 show the dynamics of this rejection from its application at 10 s.

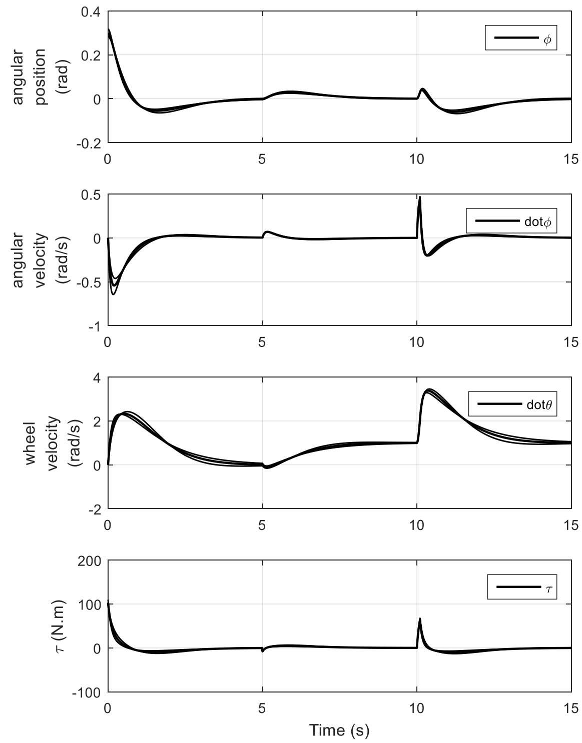

Figure 5. Closed-loop response for different values of user's mass: 60 kg, 75 kg (nominal) and 90 kg.

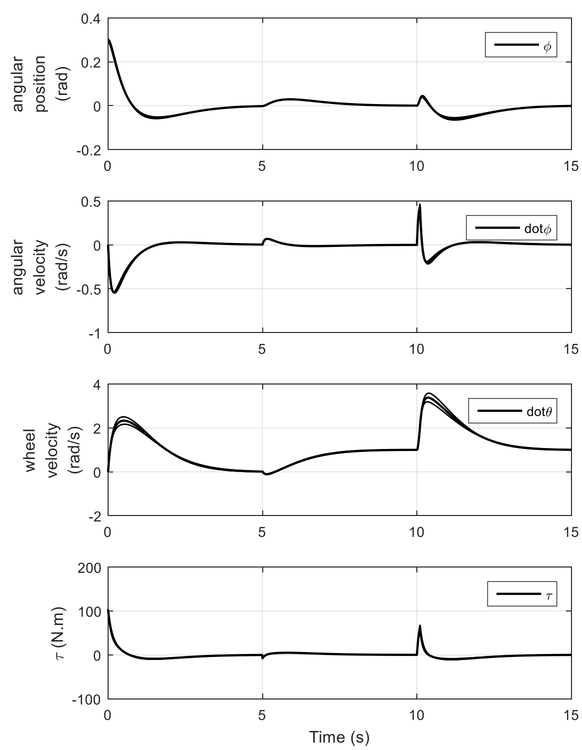

Figure 6. Closed-loop response for different values of user's stature: 1.60 m, 1.75 m (nominal) and 1.90 m.

While it is necessary to prove this hypothesis, the disturbance rejection noticed suggests that other types of disturbance can also be rejected by the control system. Such disturbance may represent, for example, small variations in floor conditions.

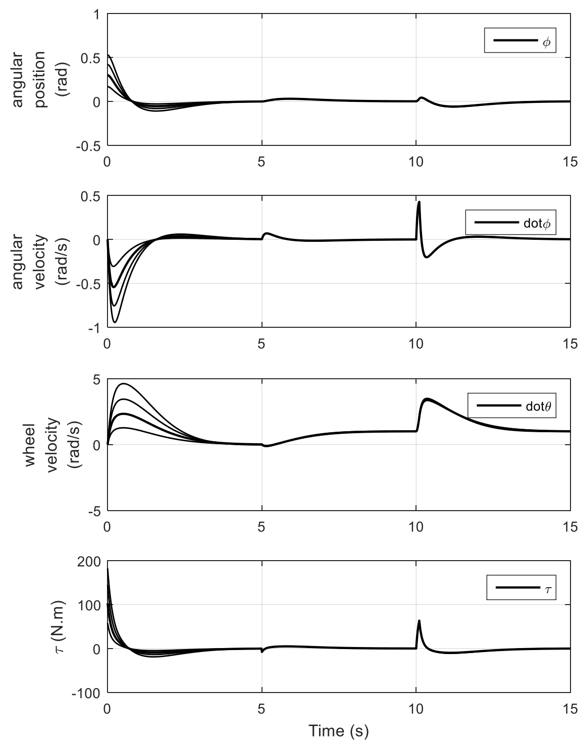

Figure 7. Closed-loop for different values of the horizontal distance of the user's shoulder with respect to rear wheel axle: -0.05 m, 0.00 m (nominal), 0.05 m and 0.10 m.

6. Discussion and Conclusion

This work investigates the control of the wheelie in power-assisted wheelchairs that to assist users in tasks such as overcoming obstacles and going down ramps. The wheelie task consists of two distinct phases: the front wheel rising from the position with all four wheels on the ground and the balance of the wheelchair-user on two wheels. The transition from the first to the second phase requires high control effort, however with short duration, while the second phase requires a closed loop control.

The proposed methodology uses a model of the wheelchair-user system whose parameters are experimentally determined for a commercial wheelchair and for an average stature and mass of the user. The lifting phase was investigated through the formulation and solution of an optimal control problem in open loop, which aims to minimize motor effort, given the physical limitations of the problem and a maximal duration of 5 s to complete this phase. The open-loop optimal control problem solution serves as a performance reference for the closed-loop controller design.

We proposed a design of the wheelchair-user system controller to maintain balance on two wheels inspired on a Linear Quadratic Regulation method for determining our state-feedback control. All references of the states were maintained null and the gains were determined by the formulation and solution of an optimal control problem in the lifting phase, without the end time specification. In order to achieve lifting patterns similar to the open-loop optimal control solution, we established restrictions on the states' settling time, whose values were obtained by trial and error until similar control and state patterns were obtained.

Aiming at practical application, no specification regarding the displacement of the wheelchair during the lift off was included in the project. This allows the setting of a speed reference by the user, for example, by means of a joystick. With this strategy, a displacement of the wheelchair during the lift off phase is expected, as shown by the θ graph in Figure 4. From a practical point view, it means the user does not have to worry about obstacles behind the wheelchair, but only in front during the lifting. Observe in Figure 4 that the lifting is almost completed in about 1 s and the remaining time is used only to fine-tune the balance. A quick lifting like this could result in discomfort to the user, but a related restriction on this could be easily incorporated into the optimal control problem, such as limiting the maximal acceleration of the φ angle.

Figures 5 to 7 illustrate the performance of the closed loop control system in three situations and parametric variations. On the one hand, the results in Figures 5 and 6 show that the controlled system response is quite insensitive to variations in mass and height of the user, regardless of the situation. On the other hand, the results in Figure 7 show large sensitivity to variations on horizontal position of the center of mass in the lift off phase, although this parameter has little effect on system response to the disturbance at t = 10 s.

The large torques at lift off, albeit with short duration, are remarkable and would require the selection of large motors. In order to reduce the size of the motors, it is possible to recommend users adjust their wheelchairs so that the center of mass is closer to the rear axle, or to instruct users to perform a trunk backwards motion just before lift off.

Figures 5 to 7 show, in the interval from 5 s to 10 s, the system response to a change in the angular velocity reference from 0 to 1 rad/s. Note that the settling time of the system velocity is of about 3 seconds with a transient response quite similar for all parametric variations considered. We notice also that at the beginning of this maneuver, the wheelchair wheels are driven briefly backwards. This sets the body in forward motion and allows initiation and subsequent sustaining of forward system motion.

Figures 5 to 7 show that system's response is insensitivity to the tested parametric variations for a short 10 N disturbance at t = 10 s. However, rejecting the disturbance requires applying a large motor torque, albeit with short duration. This means that large disturbances may cause actuator saturation, compromising rejection.

Limitations of the study include the fact that parametric robustness was evaluated for each parameter separately, representing only a response sensitivity to that particular parameter. Future studies should include the simultaneous change of multiple parameters. Another limitation of the study relates to the simplifications of the model, which include the absence of the rolling resistance force, motion constrained to a plain and not exploiting the interaction between the user and the wheelchair. Moreover, the possibility of actuator saturation was not considered. Future studies should address these limitations.

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